A novel approach for smooth contact surfaces using NURBS: application to the FE simulation of stenting

M. Stadler and G. A. Holzapfel

Institute for Structural Analysis – Computational Biomechanics
Graz University of Technology, Schiesstattgasse 14B, A-8010 Graz, Austria
e-mails: {ms|gh}@biomech.tu-graz.ac.at

Introduction

A large number of contact algorithms have been developed for standard engineering problems. Thereby, the contact surfaces are described directly by the facets of the finite elements, yielding a $C^0$ continuous contact surface. If the surface to be discretized is not flat, then the assumptions of smoothness, which are essential for the proofs of uniqueness and convexity of contact problems, are violated [3]. Therefore, if these algorithms are applied to biomechanical problems in which the surfaces are arbitrarily curved, several problems may occur: (i) in order to obtain convergence, an exceedingly fine mesh is required at the vicinity of the contact region, (ii) the error of the solution is beyond an acceptable limit, (iii) the time steps of the nonlinear solver have to be extremely small or (iv) no convergence of the solution can be obtained at all. For example, in [2] it was shown, that an error of 311% in the solution for the contact pressure is obtained for a facet-based contact simulation of an interference fit of two rings. An illustrative example of slow convergence is given in [10], where the numerical simulation of a stenting process is presented. This simulation took three weeks due to the small time steps required.

One way to avoid these problems is to overlay a smooth contact surface on the finite element mesh. Various smoothing techniques have been explored by several authors. They all have in common a continuity of $C^1$ for flexible surfaces. Higher levels of continuity have only been applied to rigid surfaces, because the applied smoothing techniques do not possess the property of local support. Hence, oscillations of the surface may occur and a large system of equations has to be solved when the smooth surface is fitted to the finite element mesh. However, the smooth surface concept of NURBS [8] provides local support. That means that only a few mesh points at the vicinity of the current finite element are involved in the fitting process which determine the shape of their associated smooth surface. This property holds for an arbitrary level of continuity.

Although in many applications of contact simulation, $C^1$ continuity seems to be sufficient, there are cases for which $C^2$ continuity is an advantage: one example is the metastable process of expanding a stent via a pressurized balloon inside an artery (see [10]). Since the applied load is deformation dependent, it is influenced by the surface curvature, which can not be controlled sufficiently by $C^1$ surfaces. The use of $C^2$ surfaces has shown a significant improvement of convergence in this case (see [9]).

Continuum mechanical formulation

Different formulations can be applied to incorporate contact constraints into the weak form. Here we have chosen the penalty method. For the derivation of the residual vector and the tangent matrix, which are needed for the finite element implementation, the Mathematica package AceGen [7] was used. It carries out the derivation with simultaneous stochastic simplification of the expressions. The resulting expressions can be exported in the formats of several compiled languages.

Search for the contact partner

The properties of NURBS can also be used for the efficient combination of two algorithms that are associated with contact detection within the master-slave concept: (i) contact partner search for a particular slave point, and (ii) identification of the point of impact in terms of the surface coordinates. The latter is necessary for the evaluation of several geometric quantities (distance, normal vector) which are required for the construction of the tangent matrix. The combination of the two algorithms - usually separated - is achieved through a special parameterization of the smooth surface. Then, a projection of a slave point onto the smooth surface, which is solved via a Newton-Raphson iteration, yields all the information that is

Fig. 1: Example of automatic surface detection with refined elements (left) and associated quad-tree (right).
required.

**Search for the neighbor element**

All smooth surface algorithms require information about the neighbor elements when building the tangent matrix for a particular finite element, since the neighbors determine the shape of the smooth surface within the current element. The operation to find these neighbors can become extremely expensive on adaptively refined meshes. An appropriate tool to avoid this is a hierarchical tree data structure (see [4] and [11]). In particular, this results in a quad-tree or oct-tree for 2D or 3D problems, respectively. Then, the neighbor search, which generally yields an algorithm of order n², where n is the number of finite elements, is replaced by a simple tree traversal. An example of this is shown in Fig. 1. Such a framework is provided by the **DEAL.II** libraries [1], which were chosen for the implementation.

**Automatic contact surface detection**

Usually, the user of a finite element processor only wants to specify the bodies coming into contact. However, most software packages require the user to specify the surfaces individually. This is tedious and error prone. The hierarchical tree data structure can be used to detect contact surfaces automatically. This is especially useful when the contacting bodies have a large number of surfaces. Since the detected contact surfaces are treated independently from the bodies, even a special case of self-contact can be simulated.

**Numerical examples**

To demonstrate the capabilities of the developed contact algorithm, we will show representative numerical examples, in which the contact stress distribution will be compared between analytical solutions and different levels of continuity of the contact surfaces. In particular we will show the pressure driven expansion of a stent during balloon angioplasty. For the simulation we have chosen a variant of the **NIR Elite** stent (**Boston Scientific**). To study the dependency of convergence rate on the level of continuity of the contact surface, we will carry out the stenting process on contact surfaces with different levels of continuity on a simplified 2D model. This is shown in Fig. 2.

![Fig. 2: (a) 2D simplification of stent deployment with an angioplasty balloon, (b) structure of the stent **NIR Elite, Boston Scientific**.](image)

We will also discuss an enhanced 3D version of the model. For the constitutive modeling as well as for the geometric modeling, which is based on high-resolution magnetic-resonance-imaging, we choose the methods which are documented in [5] and [6]. For this model we will present details of a stress intensity analysis for the modification of several geometrical parameters of arterial components towards transluminal stress development for different loading states. It is of significant clinical interest, which type of treatment of arterial plaques can contribute to their stabilization. One example is a lipid lowering therapy [12]. However, there are many geometrical quantities of arterial plaque components, whose contribution to the stress situation in the arterial wall and to plaque stability is unknown. This study may contribute to find the appropriate treatment for a particular stenosis. The examples will also demonstrate, that the results may be meaningless due to large deviations with a facet-based contact algorithm.

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**References**